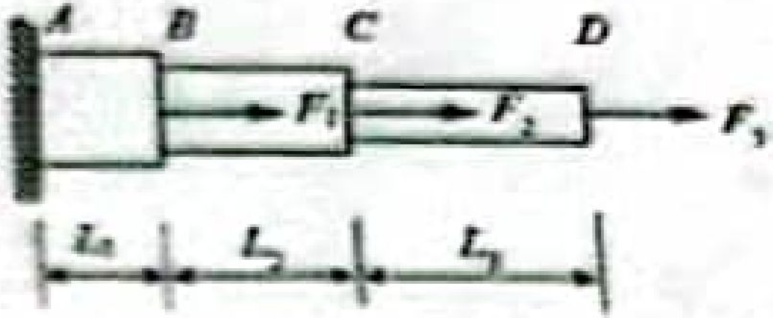
阶梯杆如图所示，从左到右三段杆的横截面面积分别为：$A_1=200\ \mathrm{mm}^2$，$A_2=150\ \mathrm{mm}^2$，$A_3=100\ \mathrm{mm}^2$。各段杆长度分别为$L_1=1$ m，$L_2=1.5$ m，$L_3=2$ m。材料的弹性模量$E=210$ GPa，轴向拉力$F_1=2$ kN，$F_2=2$ kN，$F_3=6$ kN。试：（1）作杆的轴力图；（2）求最大正应力；（3）求整根杆的伸长量。

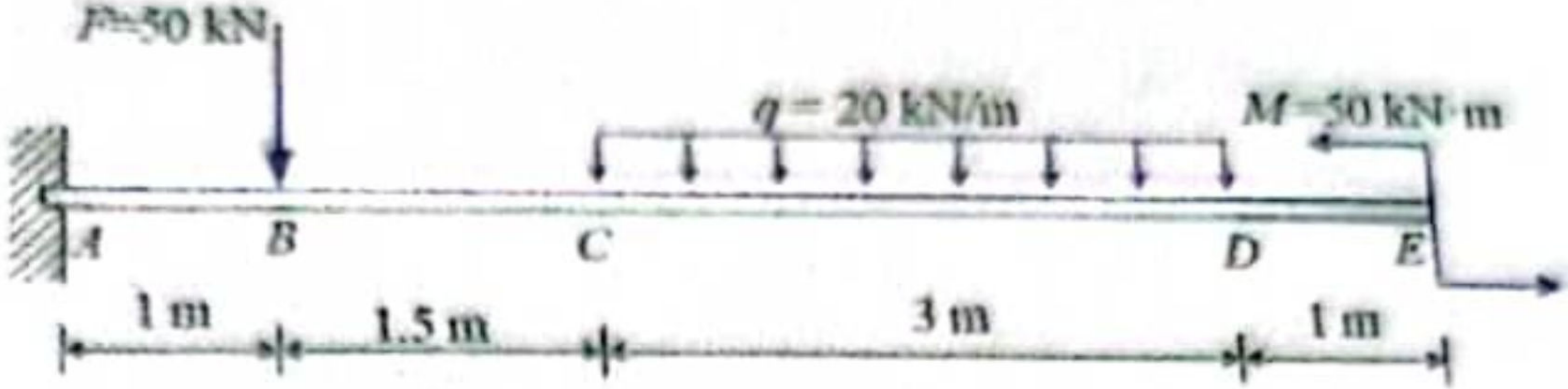


1.

计算题（10.0分）

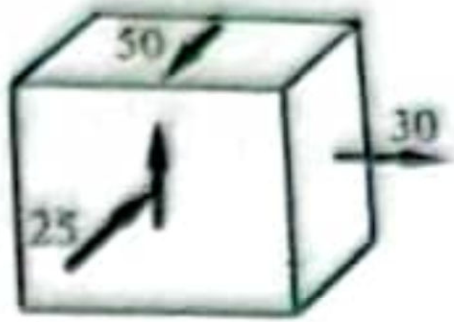
已知图示悬臂梁所受载荷 F、q、M 和尺寸，试作剪力图和弯矩图。

2.



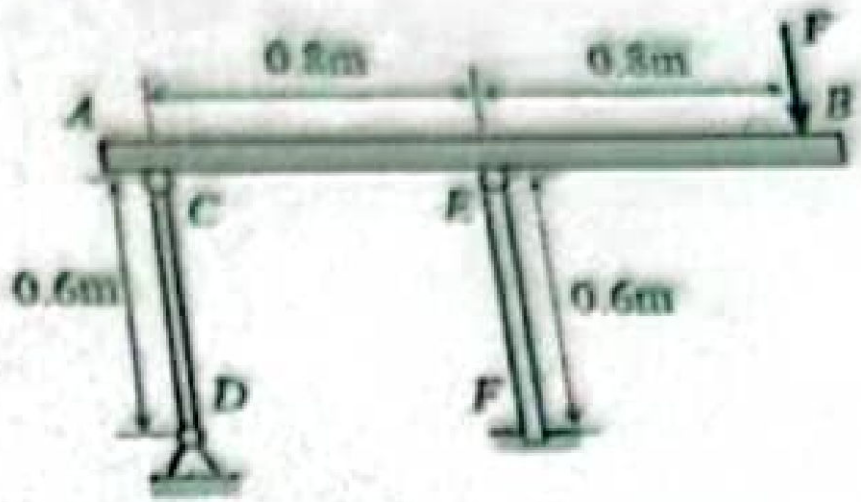
计算题（15.0分）

3\. 构件内一点的应力状态如图所示，单位为MPa，材料的弹性模量 $E = 210\ GPa$，泊松比 $\mu = 0.3$，试求：（1）主应力；（2）最大切应力；（3）最大线应变。

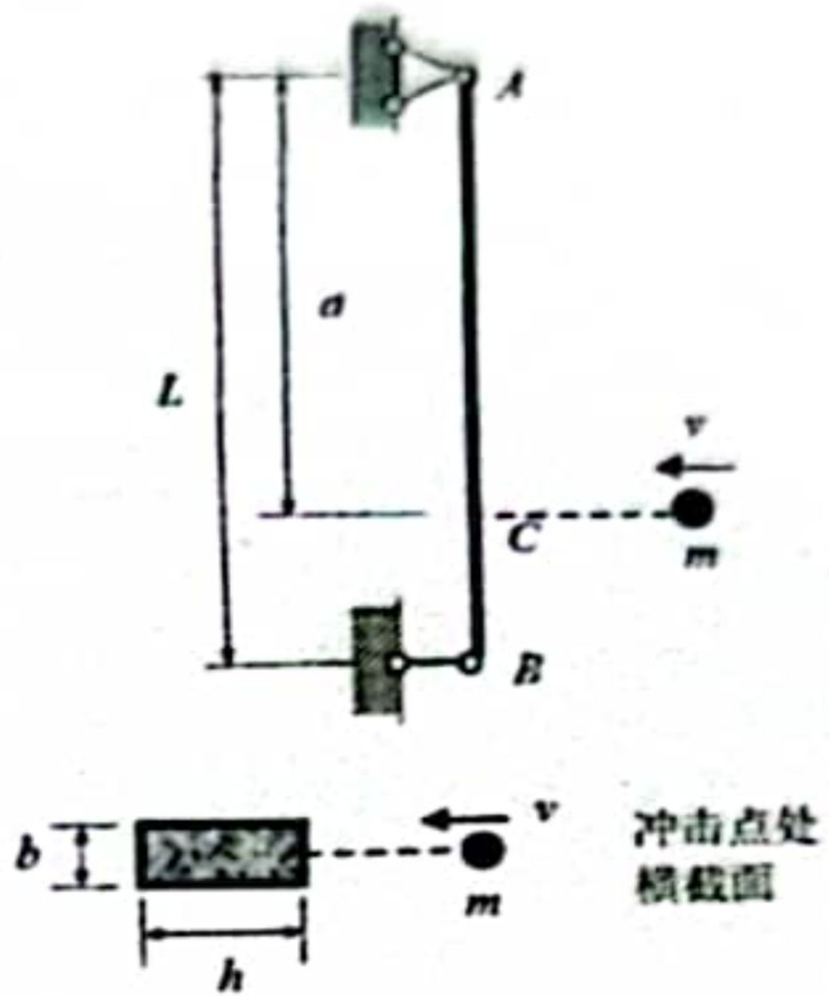


如图所示，电动机功率 $P=9\ \text{kW}$，转速 $n=715\ \text{r/min}$，带轮直径 $D=250\ \text{mm}$，已知主轴内径 $d=40\ \text{mm}$，许用应力 $[\sigma]=60\ \text{MPa}$，外伸段（AB 段）长度 $l=200\ \text{mm}$，试计算主轴危险截面上内力（扭矩、弯矩和剪力），采用第三强度理论（最大切应力理论）设计主轴外径 D'，以及计算危险点的主应力。

如图所示平面结构中，水平梁 AB 为刚性梁，作用有集中力 $F = 4$ kN，竖直杆 CD 的横截面为 5 mm × 10 mm 的矩形，竖直杆 EF 为直径 $d = 20$ mm 的圆截面杆，两杆长度均为 $l = 0.6$ m，材料相同，$E = 206$ GPa，$\sigma_p = 200$ MPa，$\sigma_s = 235$ MPa，$[\sigma] = 120$ MPa，直线经验公式中，系数 $a = 304$ MPa，$b = 1.12$ MPa，若稳定安全因数 $n_{st} = 5$，试校核结构安全性。

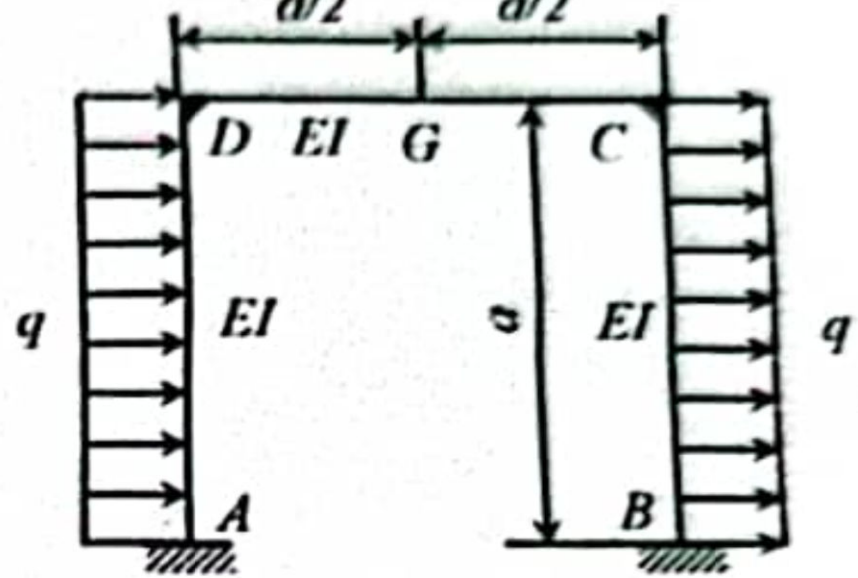


图示竖直放置的简支梁 AB，梁的长度为 L，横截面是尺寸为 $b\times h$ 的矩形，弹性模量为 E，质量不计。现有一质量为 m 的物体沿水平方向冲击该梁并附于梁上直至静止。冲击点 C 距 A 点长度为 a。接触时物体速度为 v。求水平冲击时梁中最大冲击正应力。



计算题（15.0分）

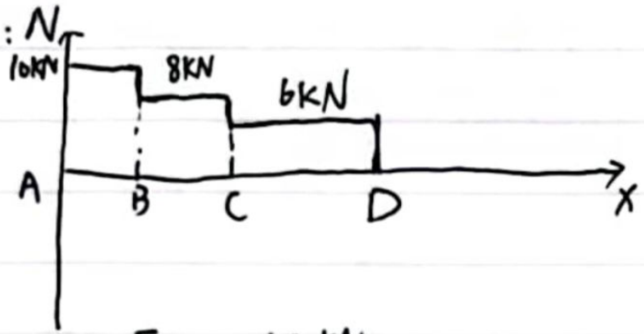
7. 图示超静定刚架 $ABCD$，A 端和 B 端固定，尺寸如图，AD 和 BC 段受均布载荷 q 作用，设刚架 EI 为常量，试用力法正则方程求对称截面（G 截面）的内力。



1\. (1) $\Sigma F_x = 0$, $-F_A + F_1 + F_2 + F_3 = 0$

$\therefore F_A = 10kN$

轴力图：



(2) AB: $\sigma_1 = \dfrac{F_N}{A_1} = \dfrac{10\,kN}{200mm^2} = 50MPa$

BC: $\sigma_2 = \dfrac{F_N}{A_2} = \dfrac{8\,kN}{150\,mm^2} = 53.3MPa$

CD: $\sigma_3 = \dfrac{F_N}{A_3} = \dfrac{6\,kN}{100\,mm^2} = 60MPa$

$\therefore$ 最大正应力为 $\sigma_3 = 60MPa$ 在CD段

(3) AB: ~~$\varepsilon = \dfrac{\sigma_1}{E}$~~ $\Delta l_1 = \varepsilon_1 l = \dfrac{\sigma_1 l}{E} = \dfrac{50MPa \times 1m}{210GPa} = 0.24mm$

BC: $\Delta l_2 = \dfrac{\sigma_2 l_2}{E} = \dfrac{53.3 \times 1.5}{210} = 0.38\ mm$

CD: $\Delta l_3 = \dfrac{\sigma_3 l_3}{E} = \dfrac{60 \times 2}{210} = 0.57\ mm$

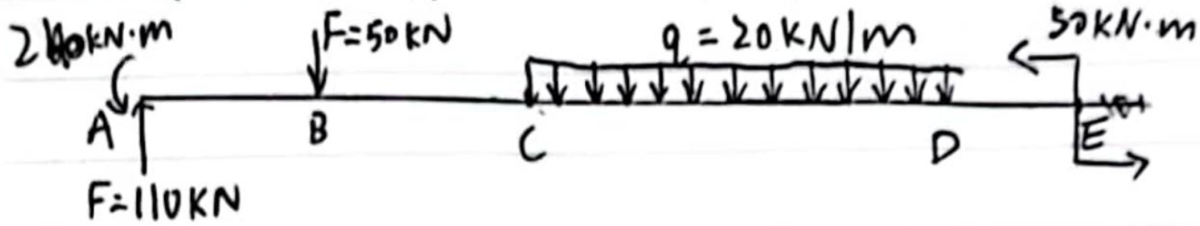
$\therefore \Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3 = 1.19\ mm$

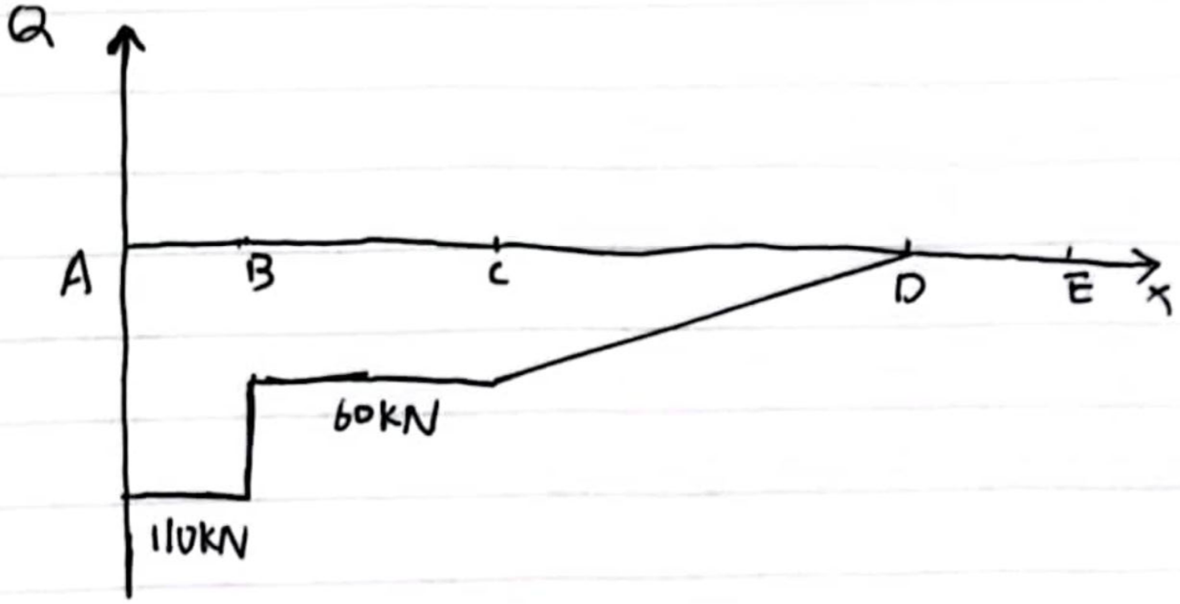
伸长量为 $\Delta L = 1.19mm$

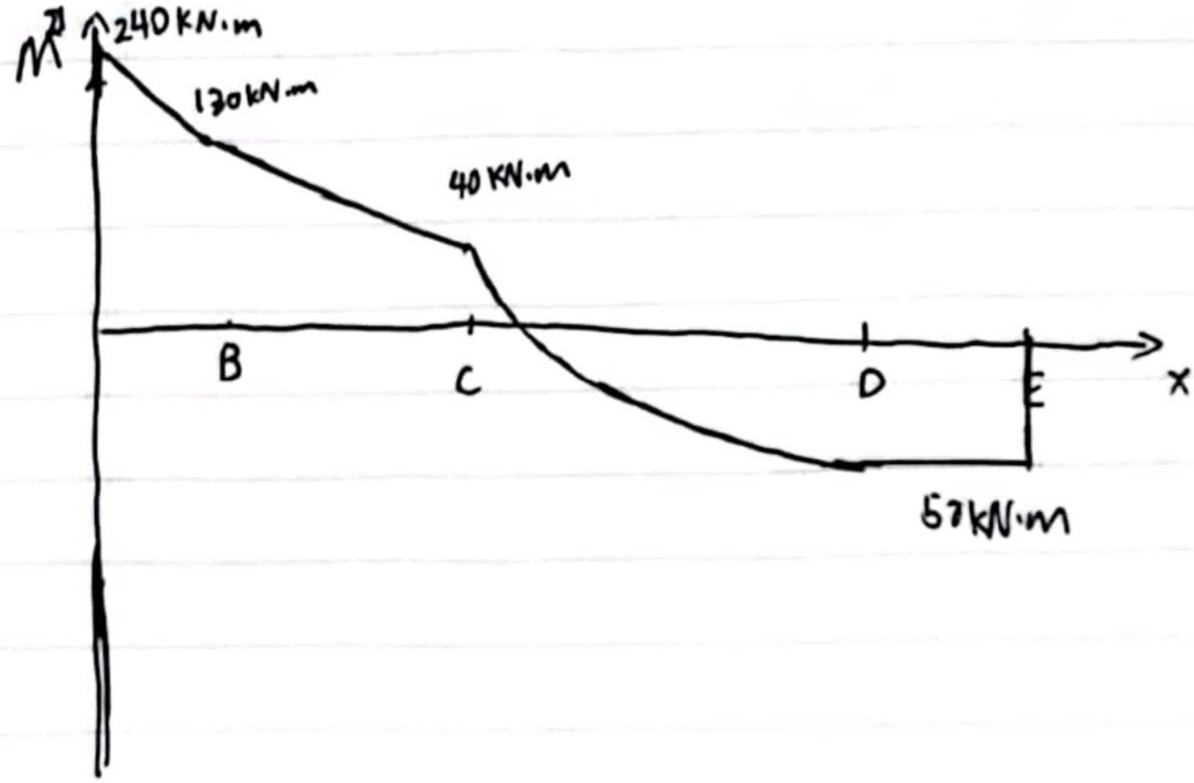
1.

$\Sigma F_y = 0,\ -F_A + F + ql = 0 \Rightarrow F_A = 110\ kN$

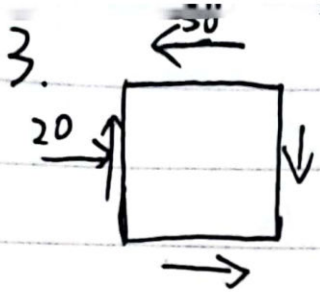
A点 $\Sigma M_A = 0,\ M_A + F\cdot 1 + ql\cdot 4 - 50 \Rightarrow M_A = 240\ kN\cdot m$







3.



(1) $\sigma_x = -20\,MPa$ $\quad \tau_{xy} = 50\,MPa$

$\sigma_y = 0$ $\quad \sigma_z = 30\,MPa$

$$\sigma_{max} = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = 40.99\,MPa$$

$$\sigma_{min} = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = -60.99\,MPa$$

$\therefore\ \sigma_1 = ~~40.99~~\ -60.99\,MPa$ $\quad \sigma_2 = 30\,MPa,$ $\quad \sigma_3 = 40.99\,MPa$

(2) $\tau_{max} = \frac{\sigma_3-\sigma_1}{2} = 50.99\,MPa$

(3) $\varepsilon_1 = \frac{1}{E}[\sigma_1 - \mu(\sigma_2+\sigma_3)] = \frac{-60.99-0.3\times(30+40.99)}{210} = -0.39$

$\varepsilon_2 = \frac{1}{E}[\sigma_2 - \mu(\sigma_1+\sigma_3)] = \frac{30-0.3\times(40.99-60.99)}{210} = 0.17$

$\varepsilon_3 = \frac{1}{E}[\sigma_3 - \mu(\sigma_1+\sigma_2)] = \frac{40.99-0.3\times(30-60.99)}{210} = 0.24$

$\therefore$ 最大线应变 $\varepsilon_3 = 0.24$

5. $\Sigma M = 0$，$F_{EF} \times 0.8 - F \times 1.6 = 0$

$\therefore F_{EF} = 8kN$

$\therefore F_{CD} = F_{EF} - F = 4kN$

$\lambda_p = \sqrt{\frac{\pi^2 E}{\sigma_p}} = 101$ $\lambda_s = a - b\sigma_s = 40.8$

$i = \sqrt{\frac{I}{A}}$ ，$\lambda = \frac{\mu l}{i}$

$I_{EF} = \frac{\pi d^4}{64} = 7854\ mm^4$ $A_{EF} = \frac{\pi d^2}{4} = 314\ mm^2$

$I_{CD} = \frac{bh^3}{12} = 417\ mm^4$ $A_{CD} = 50\ mm^2$

$\therefore \lambda_{EF} = \frac{0.7 \times 0.6 \times 10^3}{\sqrt{\frac{7854}{314}}} = 83.98 < \lambda_p$

$\lambda_{CD} = \frac{1 \times 0.6 \times 10^3}{\sqrt{\frac{417}{50}}} = 207.76 > \lambda_p$

对于 CD：$F_{cr} = \frac{\pi^2 EI}{(\mu l)^2} = \frac{\pi^2 \times 206 \times 10^3 \times 417}{(0.6 \times 10^3)^2} = 2355\ N < F_{CD}$

$\therefore$ CD不安全

对于EF：~~$F < \sigma_{EF} = \frac{F}{A} = 25.4\ MPa \leq [\sigma]$~~

$\sigma_{cr} = \frac{a - \lambda}{b} = 196\ MPa$

$\sigma_{EF} = \frac{F}{A} = \frac{8\ kN}{314\ mm^2} = 25.4\ MPa$.

$\frac{\sigma_{cr}}{n_{st}} = 39\ MPa > \sigma_{EF}$

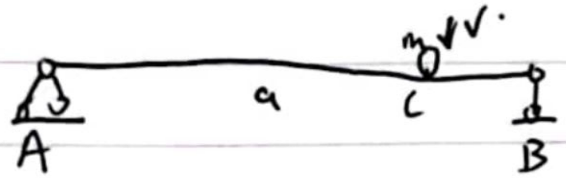
$\therefore$ EF 安全.

∴ $I_z = \frac{\quad}{12}$

设……

$K_d = 1 + \sqrt{1 + \frac{2h}{\Delta_s}}$

看成水平杆



接触时速度为 V，则 $h = \frac{V^2}{2g}$

$\Delta_s = \Delta_1 + \Delta_2$.

Δ_1 为 AC，Δ_2 为 BC

∴ 由卡二得

$$\Delta_1 = \int \frac{M(x)}{EI} \cdot \frac{\partial M(x)}{\partial F} dx = \frac{mga^3}{3EI}$$

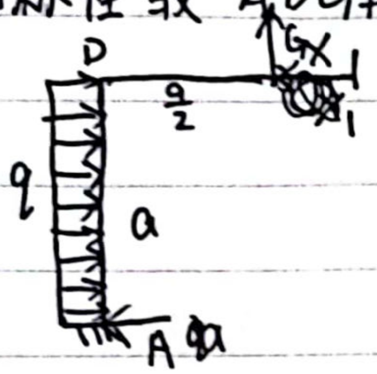
$$\Delta_2 = \int \frac{M(x)}{EI} \cdot \frac{\partial M(x)}{\partial F} dx = \frac{mg(L-a)^3}{3EI}$$

$$\therefore \Delta_s = \Delta_1 + \Delta_2 = \frac{mg}{3EI}[a^3 + (L-a)^3]$$

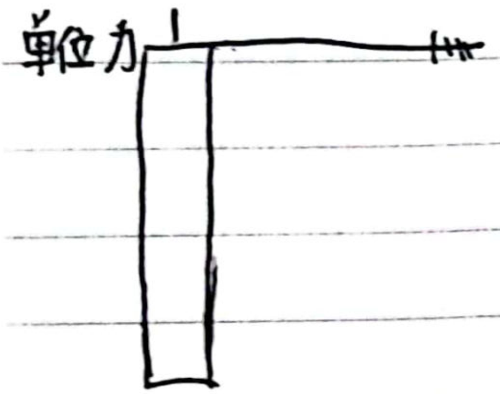
$$\therefore K_d = 1 + \sqrt{1 + \frac{\frac{V^2}{2g}}{\frac{mg[a^3 + (L-a)^3]}{3EI}}} = 1 + \sqrt{1 + \frac{3EIV^2}{mg^2[a^3 + (L-a)^3]}}$$

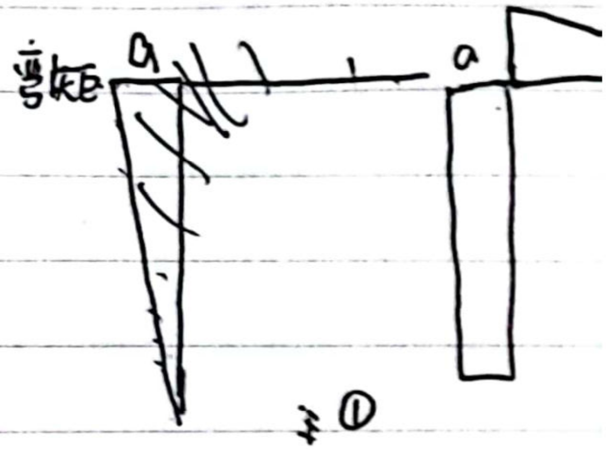
$$= 1 + \sqrt{1 + \frac{Ebh^3V^2}{4mg^2[a^3 + (L-a)^3]}}$$

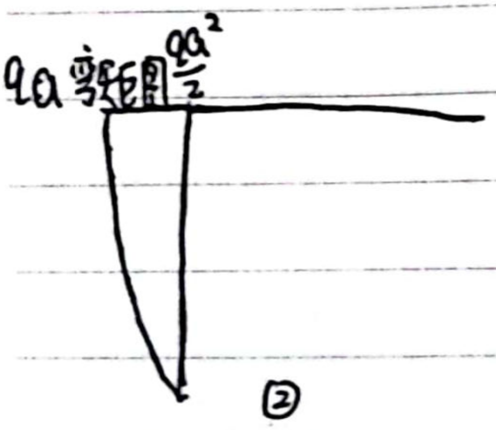
解：由对称性 取 ADG段进行分析



可列正则方程 $X_1\delta_{11}+\Delta_{1P}=0$







$$\delta_{11}=\frac{1}{E}\times\left(a\times a\times\frac{a}{2}+\frac{1}{2}a\times a\times a\right)=\frac{a^3}{E}$$

$$\Delta_{1P}=\frac{1}{E}\left(-\frac{1}{3}\times\frac{qa^2}{2}\times a\times a\right)=-\frac{qa^4}{6}$$

$$X_1=-\frac{\Delta_{1P}}{\delta_{11}}=\frac{qa}{6}$$