1. **解**：因为$\cos(\boldsymbol{n},\boldsymbol{x})=\dfrac{x}{\sqrt{x^2+y^2+z^2}}=\dfrac{3}{\sqrt{3^2+4^2+12^2}}=\dfrac{3}{13}$，所以$\theta=\arccos\dfrac{3}{13}$.□

2. **解**：因为

$$\lim_{x\to+\infty}\frac{\frac{(1+x)^{\frac{3}{2}}}{\sqrt{x}}}{x}=\lim_{x\to+\infty}\frac{(1+x)^{\frac{3}{2}}}{x^{\frac{3}{2}}}=\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^{\frac{3}{2}}=1$$

$$\lim_{x\to+\infty}\left[\frac{(1+x)^{\frac{3}{2}}}{\sqrt{x}}-x\right]=\lim_{x\to+\infty}\frac{(1+x)^{\frac{3}{2}}-x^{\frac{3}{2}}}{\sqrt{x}}=\lim_{\substack{x\to+\infty\\ x<\xi<x+1}}\frac{\frac{3}{2}\xi^{\frac{1}{2}}\cdot[(x+1)-x]}{\sqrt{x}}$$

$$=\frac{3}{2}\lim_{\substack{x\to+\infty\\ x<\xi<x+1}}\frac{\sqrt{\xi}}{\sqrt{x}}=\frac{3}{2}$$

所以斜渐近线方程为$y=x+\dfrac{3}{2}$.□

3. **解**：$S=\pi r^2=\pi(x^2+y^2)=\pi z=4\pi$.□

4. **解**：$\displaystyle\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{n+n}\right)=\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\frac{1}{1+\frac{i}{n}}=\int_0^1\frac{1}{1+x}\mathrm{d}x=\ln 2$.□

5. **解**：利用弧长公式可得

$$S=\int_a^b\sqrt{1+[y'(x)]^2}\,\mathrm{d}x=\int_0^2\sqrt{1+\left(\frac{4}{3}\cdot\frac{3}{2}x^{\frac{1}{2}}\right)^2}\,\mathrm{d}x=\int_0^2\sqrt{1+4x}\,\mathrm{d}x$$

$$=\frac{1}{6}(1+4x)^{\frac{3}{2}}\Big|_0^2=\frac{13}{3}.\square$$

6. **解**：因为

$$\lim_{x\to0}\frac{\int_0^{x^2}\sin t\,\mathrm{d}t}{x^a}=\lim_{x\to0}\frac{2x\cdot\sin x^2}{ax^{a-1}}=\lim_{x\to0}\frac{2x^3}{ax^{a-1}}=\frac{1}{2}\Longrightarrow a=4.\square$$

注：本题有误，应改为$\displaystyle\lim_{x\to0}\frac{\int_0^{x^2}\sin t\,\mathrm{d}t}{x^a}=\frac{1}{2}$.

7. **解**：$\rho=\dfrac{|f''(x)|}{\{1+[f'(x)]^2\}}=0$.□

8. **解**：直接可求得

$$F(x)=\int_0^x f(t)\mathrm{d}t=\int_0^x (t-1)\mathrm{d}t=\frac{1}{2}x^2-x,\ x>0$$

$$F(x)=\int_0^x f(t)\mathrm{d}t=\int_0^x \sin t\,\mathrm{d}t=\int_x^0 \sin t\,\mathrm{d}t=[-\cos t]\big|_x^0=\cos x-1,\ x\leqslant 0$$

$$F(x)=\begin{cases}\cos x-1,\ x\leqslant 0\\ \frac{1}{2}x^2-x,\ x>0\end{cases}\Longrightarrow F'(x)=\begin{cases}\sin x,\ x\leqslant 0\\ x-1,\ x>0\end{cases}$$

显然在$x=0$处连续不可导，选择 B.□

9. **解**：显然选择 C，B 是 C 的一种特殊情况，当且仅当$g(a)=C$的时候成立.□

10. **解**：显然 A 在$x=0$出现瑕点无定义且振动，因此发散.□

11. **解**：注意到

$$\int 3x^2\arctan x\,\mathrm{d}x=\int \arctan x\,\mathrm{d}x^3=x^3\arctan x-\int\frac{x^3}{1+x^2}\mathrm{d}x=x^3\arctan x-\frac{1}{2}\int\frac{x^2}{1+x^2}\mathrm{d}(x^2)$$

$$=x^3\arctan x-\frac{1}{2}\int\left(1-\frac{1}{1+t}\right)\mathrm{d}t=x^3\arctan x-\frac{1}{2}x^2+\frac{1}{2}\ln(1+x^2)+C.\square$$

12. **解**：注意到

$$\int\frac{1}{\sin x\cos^3 x}\mathrm{d}x=\int\frac{\sin x}{\sin^2 x\cos^3 x}\mathrm{d}x=-\int\frac{1}{(1-\cos^2 x)\cos^3 x}\mathrm{d}(\cos x)=-\int\frac{1}{(1-t^2)t^3}\mathrm{d}t$$

$$=-\int\frac{t}{(1-t^2)t^4}\mathrm{d}t=-\frac{1}{2}\int\frac{1}{(1-t^2)t^4}\mathrm{d}(t^2)=-\frac{1}{2}\int\frac{1}{(1-u)u^2}\mathrm{d}u$$

$$=-\frac{1}{2}\int\left(\frac{1}{u}+\frac{1}{u^2}+\frac{1}{1-u}\right)\mathrm{d}u=-\frac{1}{2}\left[\ln u-\frac{1}{u}-\ln(1-u)\right]$$

$$=-\frac{1}{2}\left[\ln(\cos^2 x)-\frac{1}{\cos^2 x}-\ln(1-\cos^2 x)\right]=\ln\tan x+\frac{1}{2}\sec^2 x+C.\square$$

13. **解**：注意到

$$\int_{-1}^1\frac{2x^2+\tan x}{1+\sqrt{1-x^2}}\mathrm{d}x=4\int_0^1\frac{x^2}{1+\sqrt{1-x^2}}\mathrm{d}x=4\int_0^1\left(1-\sqrt{1-x^2}\right)\mathrm{d}x=4-4\int_0^1\sqrt{1-x^2}\,\mathrm{d}x$$

$$=4-4\cdot\frac{1}{4}\pi\cdot 1^2=4-\pi.\square$$

14. **解**：注意到

$$F(x)=\int_{-1}^x f(t)\mathrm{d}t=\int_{-1}^x\frac{3}{2}t^2\mathrm{d}t=\frac{1}{2}t^3\Big|_{-1}^x=\frac{1}{2}x^3+\frac{1}{2},\ -1\leqslant x<0$$

$$F(x)=\int_{-1}^x f(t)\mathrm{d}t=\int_{-1}^0\frac{3}{2}t^2\mathrm{d}t+\int_0^x\frac{\mathrm{e}^t}{\mathrm{e}^t+1}\mathrm{d}t=\frac{1}{2}+\ln(\mathrm{e}^t+1)\big|_0^x$$

$$=\frac{1}{2}+\ln(\mathrm{e}^x+1)-\ln 2,\ 0\leqslant x\leqslant 1$$

因此可得

$$F(x)=\begin{cases}\dfrac{1}{2}x^3+\dfrac{1}{2}, & -1\leqslant x<0\\ \dfrac{1}{2}+\ln(e^x+1)-\ln 2, & 0\leqslant x\leqslant 1\end{cases}.\square$$

15. **解**：注意到

$$\lim_{a\to+\infty}\int_a^{a+1}\frac{\sqrt{x}}{\sqrt{x+\sin x}}dx=\lim_{\substack{a\to+\infty\\ a<\xi<a+1}}\frac{\sqrt{\xi}}{\sqrt{\xi+\sin\xi}}[(a+1)-a]$$
$$=\lim_{\substack{a\to+\infty\\ a<\xi<a+1}}\frac{\sqrt{\xi}}{\sqrt{\xi+\sin\xi}}=1.\square$$

16. **解**：注意到

$$\int_0^1 f(x)dx=\int_0^1 dx\int_1^x\frac{\ln(t+1)}{t}dt=-\int_0^1 dx\int_x^1\frac{\ln(t+1)}{t}dt=-\int_0^1 dt\int_0^t\frac{\ln(t+1)}{t}dx$$
$$=-\int_0^1\ln(t+1)dt=-[(t+1)\ln(t+1)-t]\Big|_0^1=1-2\ln 2.\square$$

注：本题还可以利用分部积分求解.

17. **解**：设过原点 O 的平面为 $Ax+By+Cz=0$，因为 $A=(6,-3,2)$ 在平面上，且该平面与

$4x-y+2z-8=0$ 垂直，所以有

$$\begin{cases}6A-3B+2C=0\\ 4A-B+2C=0\end{cases}\Longrightarrow\begin{cases}A=A\\ B=A\\ C=-\dfrac{3}{2}A\end{cases}$$

由此可得平面方程为

$$x+y-\frac{3}{2}z=0.\square$$

18. **解**：(1)直接利用旋转体公式可得

$$V=\pi\int_a^b f^2(x)dx=\pi\int_{-1}^1(1-x^2)^2dx=2\pi\int_0^1(1-x^2)^2dx\xlongequal{x=\cos t}2\pi\int_{\frac{\pi}{2}}^0(1-\cos^2t)^2d(\cos t)$$
$$=2\pi\int_0^{\frac{\pi}{2}}\sin^5t\,dt=2\pi\cdot\frac{4}{5}\cdot\frac{2}{3}=\frac{16}{15}\pi.\square$$

(2)直接利用旋转体公式可得

$$V=\pi\int_a^b x^2(y)dy=\pi\int_0^1\left(\sqrt{1-y}\right)^2dy=\pi\int_0^1(1-y)dy=\frac{\pi}{2}.\square$$

19. **解**：设$A=\int_0^{\frac{\pi}{4}} f(x)\sec^2 x\,\mathrm{d}x$，注意到

$$A=\int_0^{\frac{\pi}{4}} x\sec^2 x\,\mathrm{d}x - A\int_0^{\frac{\pi}{4}}\sec^2 x\,\mathrm{d}x = x\tan x\Big|_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}}\tan x\,\mathrm{d}x - A\tan x\Big|_0^{\frac{\pi}{4}}$$
$$=\frac{\pi}{4}-\int_0^{\frac{\pi}{4}}\tan x\,\mathrm{d}x - A = \frac{\pi}{4}+\ln(\cos x)\Big|_0^{\frac{\pi}{4}} - A = \frac{\pi}{4}-\frac{1}{2}\ln 2 - A$$

由此可得$A=\frac{\pi}{8}-\frac{\ln 2}{4}$，$f(x)=x+\frac{\ln 2}{4}-\frac{\pi}{8}$.□

20. **解**：略，见书.

21. **解**：注意到

$$\int_a^b f(x)\mathrm{d}x-(b-a)f\left(\frac{a+b}{2}\right)=\int_a^b\left[f(x)-f\left(\frac{a+b}{2}\right)\right]\mathrm{d}x$$
$$=\int_a^b\left[f\left(\frac{a+b}{2}\right)+\left(x-\frac{a+b}{2}\right)f'\left(\frac{a+b}{2}\right)+\left(x-\frac{a+b}{2}\right)^2\frac{f''(\xi)}{2}-f\left(\frac{a+b}{2}\right)\right]\mathrm{d}x$$
$$=f'\left(\frac{a+b}{2}\right)\int_a^b\left(x-\frac{a+b}{2}\right)\mathrm{d}x+\frac{f''(\xi)}{2}\int_a^b\left(x-\frac{a+b}{2}\right)^2\mathrm{d}x$$
$$\xlongequal{x=\frac{a+b}{2}+\frac{b-a}{2}t}\frac{f''(\xi)}{2}\left(\frac{b-a}{2}\right)^3\int_{-1}^1 t^2\,\mathrm{d}t=\frac{(b-a)^3}{24}f''(\xi)\geqslant 0.\square$$

(法二)注意到$f''(x)>0$，所以$f(x)$是下凹的，即在区间$[a,b]$上的切线均在曲线$f(x)$下，即有

$$f'\left(\frac{a+b}{2}\right)\left(x-\frac{a+b}{2}\right)+f\left(\frac{a+b}{2}\right)\leqslant f(x)$$
$$\int_a^b\left[f'\left(\frac{a+b}{2}\right)\left(x-\frac{a+b}{2}\right)+f\left(\frac{a+b}{2}\right)\right]\mathrm{d}x\leqslant\int_a^b f(x)\mathrm{d}x$$
$$(b-a)f\left(\frac{a+b}{2}\right)\leqslant\int_a^b f(x)\mathrm{d}x.\square$$